A long, rectangular metallic blade has width L= 4 cm and temperature $T_w = 40^0$C. It is surrounded on both sides by atmospheric air at $T_\infty = 20^0$C. The long side of the blade is always horizontal. Calculate the total heat transfer rate per unit of blade length when the short side of its rectangular shape (L) is (a) vertical, (b) inclined at 45^0 relative to the vertical, and (c) horizontal. Comment on the effect that blade orientation has on the total heat transfer rate.

In[1]:= $$Nu = \left(0.825 + \frac{0.387\, Ra_L^{1/6}}{\left(1 + \left(\frac{0.492}{Pr}\right)^{9/16}\right)^{8/27}}\right)^2;$$

In[2]:= $$Ra_L = \frac{g \operatorname{Cos}[\theta]\, \beta\, (T_w - T_\infty)\, L^3}{\nu\, \alpha};$$

In[3]:=
```
SetDirectory[NotebookDirectory[]];
airProps = Import["../air_props.csv"];
airProps⟦2 ;;, 5⟧ = airProps⟦2 ;;, 5⟧ 10^-3;
airProps⟦2 ;;, 7⟧ = airProps⟦2 ;;, 7⟧ 10^-6;
airProps⟦2 ;;, 8⟧ = airProps⟦2 ;;, 8⟧ 10^-6;
νI = Interpolation[airProps⟦2 ;;, {1, 7}⟧];
βI = Interpolation[airProps⟦2 ;;, {1, 5}⟧ ];
PrI = Interpolation[airProps⟦2 ;;, {1, 9}⟧];
αI = Interpolation[airProps⟦2 ;;, {1, 8}⟧];
κI = Interpolation[airProps⟦2 ;;, {1, 4}⟧];
```

In[13]:= $$TFilm = \frac{T_w + T_\infty}{2};$$

```
propertyVals = {ν → νI[TFilm], α → αI[TFilm],
   β → βI[TFilm], Pr → PrI[TFilm], κ → κI[TFilm], g → 9.81};
```

$$h = Nu \frac{\kappa}{L};\; q = h\, L\, (T_w - T_\infty);$$

In[16]:= partA = {T_w → 40, T_∞ → 20, L → 0.04, θ → 0 Degree};

In[17]:= h /. propertyVals // # /. partA &

Out[17]=

6.28886

In[18]:= qPartA = 2 q /. propertyVals // # /. partA &

Out[18]=

10.0622

In[19]:= partB = {T_w → 40, T_∞ → 20, L → 0.04, θ → 45 Degree};

In[20]:= Ra_L /. propertyVals // # /. partB &

Out[20]=

81 015.9

In[21]:= qPartB = 2 q /. propertyVals // # /. partB &

Out[21]=

9.2516

In[30]:= $$Ra_{L_c} = \frac{g\, \beta\, (T_w - T_\infty)\, L_c^3}{\nu\, \alpha};$$

$$Nu_{top} = 0.59\, Ra_{L_c}^{1/4};\; Nu_{bottom} = 0.27\, Ra_{L_c}^{1/4};$$

In[32]:=
$$h_{top} = Nu_{top} \frac{\kappa}{L_c};\ h_{bottom} = Nu_{bottom} \frac{\kappa}{L_c};$$
$$L_c = \text{Limit}\left[\frac{L\,W}{2\,(L+W)}, W \to \infty\right];$$
$$q_{top} = h_{top}\, L\, (T_w - T_\infty);\ q_{bottom} = h_{bottom}\, L\, (T_w - T_\infty);$$
$$q_{total} = q_{top} + q_{bottom};$$

In[36]:=
$$\text{partC} = \{T_w \to 40,\ T_\infty \to 20,\ L \to 0.04,\ \theta \to 0\ \text{Degree}\};$$

In[37]:=
$$\text{qPartC} = q_{total}\ \text{/. propertyVals // \# /. partC \&}$$

Out[37]=

9.9372

In[38]:=
```
Grid[{{"Vertical", "45⁰ inclined", "Horizontal"},
  {qPartA, qPartB, qPartC}}, Frame → All]
```

Out[38]=

Vertical	45^0 inclined	Horizontal
10.0622	9.2516	9.9372