In[•]:=
```
v = v0[y] (x / δ[y]) (1 - x / δ[y]) ^ 2
```
Out[•]=

$$\frac{x\, v0[y] \left(1 - \frac{x}{\delta[y]}\right)^2}{\delta[y]}$$

In[•]:=
```
dT = qw δ[y] / κ
```
Out[•]=

$$\frac{qw\, \delta[y]}{\kappa}$$

In[•]:=
```
T = T0 + dT (1 - x / δ[y]) ^ 2
```
Out[•]=

$$T0 + \frac{qw \left(1 - \frac{x}{\delta[y]}\right)^2 \delta[y]}{\kappa}$$

In[•]:=
```
momInt = ∂y ∫0^δ[y] v^2 ⅆx == -ν (∂x v /. x → 0) + ∫0^δ[y] g β (T - T0) ⅆx
```
Out[•]=

$$\frac{2}{105} v0[y] \times \delta[y]\, v0'[y] + \frac{1}{105} v0[y]^2 \delta'[y] == -\frac{\nu\, v0[y]}{\delta[y]} + \frac{g\, qw\, \beta\, \delta[y]^2}{3\, \kappa}$$

In[•]:=
```
egyInt = ∂y ∫0^δ[y] v (T - T0) ⅆx == -α (∂x T /. x → 0) // Simplify[#, qw / κ > 0] &
```
Out[•]=

$$60\, \alpha == \delta[y]\, (\delta[y]\, v0'[y] + 2\, v0[y]\, \delta'[y])$$

In[•]:=
```
rules = {
  v0[y] → A y^m,
  v0'[y] → D[A y^m, y],
  δ[y] → B y^n,
  δ'[y] → D[B y^n, y]
 }
```
Out[•]=

$$\left\{v0[y] \to A\, y^m,\ v0'[y] \to A\, m\, y^{-1+m},\ \delta[y] \to B\, y^n,\ \delta'[y] \to B\, n\, y^{-1+n}\right\}$$

In[•]:=
```
eq1 = momInt /. rules
```
Out[•]=

$$\frac{2}{105} A^2\, B\, m\, y^{-1+2\, m+n} + \frac{1}{105} A^2\, B\, n\, y^{-1+2\, m+n} == \frac{B^2\, g\, qw\, y^{2\, n}\, \beta}{3\, \kappa} - \frac{A\, y^{m-n}\, \nu}{B}$$

In[•]:=
```
eq2 = (egyInt /. rules)
```
Out[•]=

$$60\, \alpha == B\, y^n \left(A\, B\, m\, y^{-1+m+n} + 2\, A\, B\, n\, y^{-1+m+n}\right)$$

Comparing the powers of y since above equation has to be true for all values of y

In[•]:=
```
mnSol = Solve[{2 m + n - 1 == 2 n, 2 n == m - n, 0 == m + 2 n - 1}, {m, n}]
```
Out[•]=

$$\left\{\left\{m \to \frac{3}{5},\ n \to \frac{1}{5}\right\}\right\}$$

In[•]:=
```
ABSol = Solve[{
    Simplify[eq1 /. mnSol, y > 0] // (#[[1]] &),
    Simplify[eq2 /. mnSol, {y > 0, Tw - T0 > 0}] // (#[[1]] &)},
    {A, B}]
```

Out[•]=

$$\left\{\left\{A \to \frac{10\ (-6)^{1/5}\ g^{2/5}\ qw^{2/5}\ \beta^{2/5}\ (\alpha\ \kappa\ (4\ \alpha + 5\ \nu))^{3/5}}{-4\ \alpha\ \kappa - 5\ \kappa\ \nu},\ B \to \frac{(-6)^{2/5}\ \left(4\ \alpha^2\ \kappa + 5\ \alpha\ \kappa\ \nu\right)^{1/5}}{g^{1/5}\ qw^{1/5}\ \beta^{1/5}}\right\},\right.$$

$$\left\{A \to \frac{10 \times 6^{1/5}\ g^{2/5}\ qw^{2/5}\ \beta^{2/5}\ (\alpha\ \kappa\ (4\ \alpha + 5\ \nu))^{3/5}}{4\ \alpha\ \kappa + 5\ \kappa\ \nu},\ B \to \frac{6^{2/5}\ \left(4\ \alpha^2\ \kappa + 5\ \alpha\ \kappa\ \nu\right)^{1/5}}{g^{1/5}\ qw^{1/5}\ \beta^{1/5}}\right\},$$

$$\left\{A \to \frac{10\ (-1)^{3/5}\ 6^{1/5}\ g^{2/5}\ qw^{2/5}\ \beta^{2/5}\ (\alpha\ \kappa\ (4\ \alpha + 5\ \nu))^{3/5}}{-4\ \alpha\ \kappa - 5\ \kappa\ \nu},\right.$$

$$\left.B \to -\frac{(-1)^{1/5}\ 6^{2/5}\ \left(4\ \alpha^2\ \kappa + 5\ \alpha\ \kappa\ \nu\right)^{1/5}}{g^{1/5}\ qw^{1/5}\ \beta^{1/5}}\right\},$$

$$\left\{A \to \frac{10\ (-1)^{4/5}\ 6^{1/5}\ g^{2/5}\ qw^{2/5}\ \beta^{2/5}\ (\alpha\ \kappa\ (4\ \alpha + 5\ \nu))^{3/5}}{4\ \alpha\ \kappa + 5\ \kappa\ \nu},\right.$$

$$\left.B \to -\frac{(-1)^{3/5}\ 6^{2/5}\ \left(4\ \alpha^2\ \kappa + 5\ \alpha\ \kappa\ \nu\right)^{1/5}}{g^{1/5}\ qw^{1/5}\ \beta^{1/5}}\right\},$$

$$\left\{A \to \frac{10\ (-1)^{2/5}\ 6^{1/5}\ g^{2/5}\ qw^{2/5}\ \beta^{2/5}\ (\alpha\ \kappa\ (4\ \alpha + 5\ \nu))^{3/5}}{4\ \alpha\ \kappa + 5\ \kappa\ \nu},\right.$$

$$\left.\left.B \to \frac{(-1)^{4/5}\ 6^{2/5}\ \left(4\ \alpha^2\ \kappa + 5\ \alpha\ \kappa\ \nu\right)^{1/5}}{g^{1/5}\ qw^{1/5}\ \beta^{1/5}}\right\}\right\}$$

In[•]:=
```
ASol = Simplify[ABSol[[2]][[1]][[2]]]
```

Out[•]=

$$\frac{10 \times 6^{1/5}\ g^{2/5}\ qw^{2/5}\ \beta^{2/5}\ (\alpha\ \kappa\ (4\ \alpha + 5\ \nu))^{3/5}}{4\ \alpha\ \kappa + 5\ \kappa\ \nu}$$

In[•]:=
```
BSol = Simplify[ABSol[[2]][[2]][[2]]]
```

Out[•]=

$$\frac{6^{2/5}\ (\alpha\ \kappa\ (4\ \alpha + 5\ \nu))^{1/5}}{g^{1/5}\ qw^{1/5}\ \beta^{1/5}}$$

In[•]:=
```
Tw = T /. {x → 0, δ[y] → BSol y^(1 / 5)} // # /. g → Ra ν α κ / (β qw y^4) & //
    # /. ν → Pr α & // Simplify[#, {qw > 0, κ > 0, α > 0, y > 0, β > 0}] &
```

Out[•]=

$$T0 + \frac{6^{2/5}\ (4 + 5\ Pr)^{1/5}\ qw\ y}{(Pr\ Ra)^{1/5}\ \kappa}$$

In[•]:=
```
Tw - T0 // N[#] &
```

Out[•]=

$$\frac{2.04767\ (4. + 5.\ Pr)^{1/5}\ qw\ y}{(Pr\ Ra)^{1/5}\ \kappa}$$